Kepler’s Laws of Planetary Motion

Kepler’s Laws of Planetary Motion

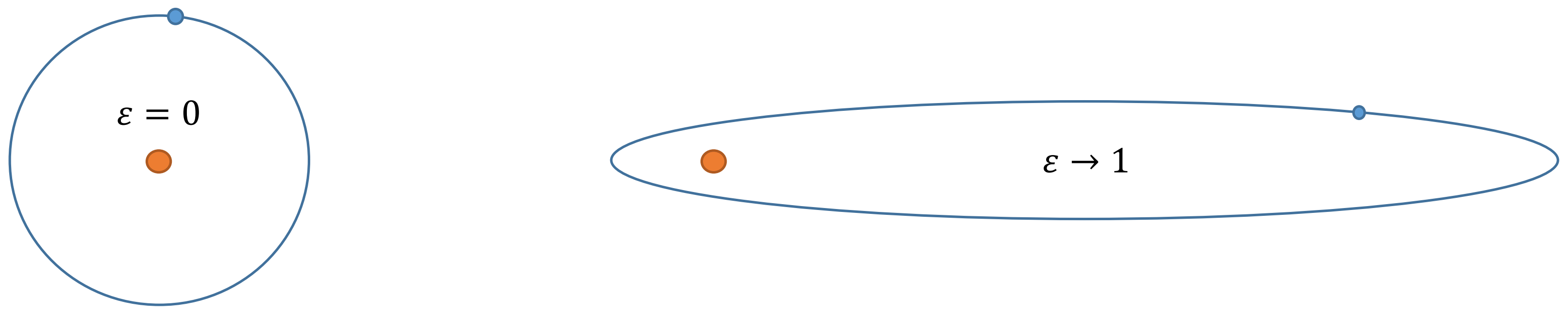
Kepler’s laws of planetary motion describe the motion and basic orbital mechanics of two point source masses.

In our Solar System there are 8 planets orbiting around one mass – the Sun. As the mass of the Sun is ~98% the mass of the entire Solar System, the gravitational forces on planets due to the mass of the other planets is negligible. Therefore, when investigating the gravitational effects between the Sun and an orbiting planet, the scenario can be treated mathematically as two point source masses.

1. The Law of Orbits

Kepler's 1st law: All planets in our Solar System orbit the Sun in an elliptical shape (the intensity of the elliptical shape is denoted by eccentricity ε) with the Sun at one focus. Where $\varepsilon = 0$ is a circular orbit and $\varepsilon = 1$ is a parabolic orbit.

So, for an elliptical orbit: $0 < \varepsilon < 1$ with eccentricity increasing towards 1.



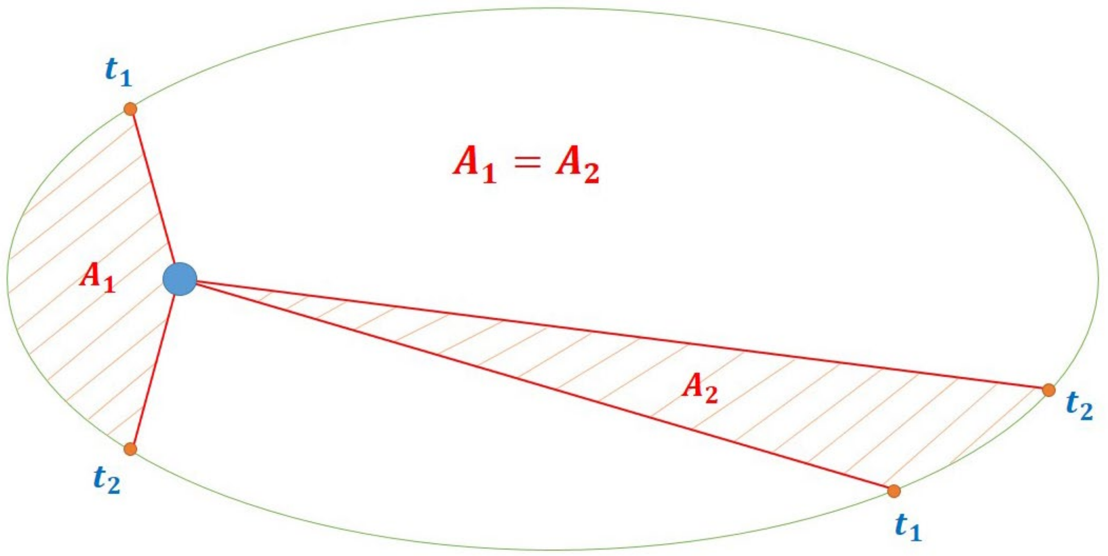
1. The Law of Orbits

The planets in our solar system orbit the Sun in a shape close to that of a circle, which means their eccentricities (ε) are very low (whereas for bodies such as comets, it can vary up to very high eccentricities).

Planet	Eccentricity ε	Planet	Eccentricity ε
Mercury	0.2056	Jupiter	0.0485
Venus	0.0068	Saturn	0.0556
Earth	0.0167	Uranus	0.0472
Mars	0.0934	Neptune	0.0086
		Pluto	0.25

2. The Law of Areas

Kepler's 2nd Law: A line drawn from the centre of the Sun to the centre of an orbiting body will sweep out equal areas in equal intervals of time, in this case with a high eccentricity to demonstrate the concept.

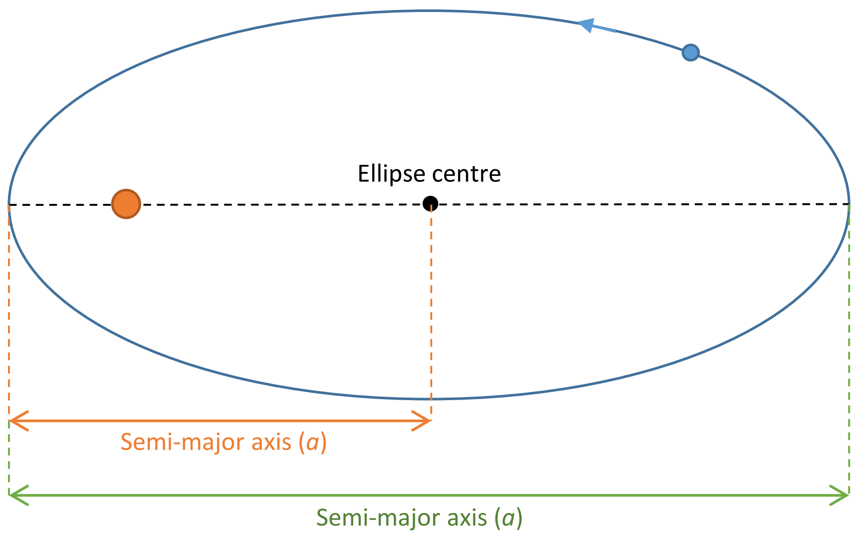


As an orbiting object moves closer to the centre of mass it is orbiting, it speeds up. At the orbiting object's farthest point from the body (aphelion) is when it is at its slowest. Thus, an object will travel a larger distance when it is at its closest point to the centre of mass (perihelion) than when it is at its farthest.

3. The Law of Periods

Kepler's 3rd law: The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their semi-major axis *(a)* of their elliptical orbit.

The semi-major axis (*a*) is half of the major axis of an ellipse i.e. the longest diameter of an ellipse:



$$T^2 = \frac{4\pi^2}{GM}a^3$$

Kepler's 3rd Law

3. The Law of Periods

In the Solar System scenario, we have seen that the planets have close to circlular ellipses with eccentricities, ε, close to 0. This means that the above equation can be slightly simplified, as the semi-major axis of the ellipse is the same as the average distance from the Sun of a circular planetary orbit:

$$T^2 = \frac{4\pi^2}{GM_*} r^3$$

3. The Law of Periods

Using the EXCEL spreadsheet, investigate the relationship of the Law of Periods equation by filling in the missing numbers and plotting T^2 against r^3.

Planetary data can be found on the NSO website (distance from the Sun, orbital period etc).

The unit of 1 AU (Astronomical Unit, the distance from the Earth to the Sun) is given to you in the spreadsheet along with other useful constants.

3. The Law of Periods

Attach a line of best fit to your plot. What does the line tell you?

The Mass of the Sun

Given that we now have all of the variables for each planet,

rearrange $T^2 = \frac{4\pi^2}{GM_*} r^3$ to make M_* (the mass of the Sun)

the subject and fill in for each planet in the Solar System.

Note: For this you will be using $G = 6.67 \times 10^{-11}\, m^3\, kg^{-1} s^{-2}$

so be careful with your units (you may have chosen to work in units of AU so far..)

The Mass of the Sun

All of your values for the mass of the Sun should be relatively similar and close to the accepted value of $1.989 \times 10^{30} kg$.

Are there any planets that differ from this value more than others? Why?

Investigating other Stellar Systems

At this point, we have used Kepler's 3rd law to calculate the mass of the Sun in kilograms.

The method that we have used is a completely viable way of using exoplanet orbits to calculate/verify the masses of central stars in a stellar system, but, as calculating the mass of stars is relatively easy using temperature, luminosity and characteristics of stellar evolution, we will use stars with known masses to calculate the distances to their respective exoplanets.

This only requires using the same equation as before but this time we are looking for *r*.

Investigating TRAPPIST-1 Exoplanets

In 2017, the Liverpool Telescope was involved in the investigation of an exciting new discovery; the star system TRAPPIST-1, which is believed to have a number of orbiting planets in habitable zones around the star.

For context, Earth lies in our Solar System’s habitable zone for life as we know it to evolve; the distance of the Earth from the Sun means that the temperature is not too hot or too cold and for which a planet has sufficient atmospheric pressure and surface conditions to support liquid water.

Investigating TRAPPIST-1 Exoplanets

Using the second EXCEL tab within the spreadsheet, calculate the distances of the TRAPPIST-1 system's planets to the host star.

- Open up the second tab of the spreadsheet labelled 'TRAPPIST-1'.
- Use the table provided to calculate the distances from the system's star to its orbiting planets.
- *There is no need to modify the graph in this spreadsheet, this is simply an aid and should provide a similar trendline/line of best fit to the one you produced earlier.*

Investigating TRAPPIST-1 Exoplanets

Check your results online to see if you have calculated the correct distances.

Remember, the 'distances' are actually called the 'semi-major axis'.

- How close were your results to the accepted values?
- Was there any potential cause of error with your calculations or anything that we did not consider?
- Are there any other astronomical scenarios you can think of where the laws of Kepler will apply?